Problem Tutorial: "Angle Beats"

Let's create graph for general graph matching.

For each "*" or "+" we will have two connected vertices and for each "." we will have only one vertex.

If the symbol is "*", we will connect one copy of the vertex with each horizontal neighbor ".", and another copy to each vertical neighbor "." . If the symbol is "+", we will connect both copies with all adjacent dots.

Let's find maximum cardinalty matching in this graph. If both copies of the vertex are connected to some adjacent vertex, let's add this tromino to answer.

Problem Tutorial: "Best Subsequence"

There are several possible solutions, intendent one is:

Obviously, minimum always will be in answer. So, let's cyclically shift our array, so that the first element will be minimum. Now, let's use binary search, and inside it we will have LIS-like dp, to find longest possible good Subsequence.

Problem Tutorial: "Cool Pairs"

There is an answer for each k. To find it, let's find the two pointers of each permutation, for each i you need to find largest j such that $a_{p_i} + b_{q_j} < 0$. You can find these pointers greedily from the left, each time pushing down all pointers to the right, and checking that the maximum number of good pairs is at least k.

Problem Tutorial: "Dates"

To find largest weight matching, let's sort all girls by weight (from the largest), then you need to add girl to the answer and check that the matching is possible. (you can prove this algorithm with Rado-Edmonds algorithm for matroids).

How to check that the matching is possible? The problem H is a hint (not intended one :)), yes, we will use hall theorem, and to check that we can find the perfect matching we will check the constraints of Hall's theorem for Arabian countries with segment tree.

We need to check that for each chosen pair of segments, if we will consider them in the given order, $j - i$ should be at most $a_{l_i} + a_{l_i+1} + \ldots a_{r_j}$.

Let's keep largest $ind_j - psums_{r_j}$ and smallest $ind_i - psums_{l_i}$ in the vertex of segment tree, now all what you need to do is to check the inversions of these values of two children in each vertex.

Problem Tutorial: "Expected Value"

Let's find smallest linear recurrence for p_i — probability of that you will be in vertex n after i seconds. You can find it with Berlekamp-Massey.

It is a linear recurrence, because each next value is a multiplication of vector by matrix, and by Hamilton-Caley theorem obviously one value of a vector is a linear recurrence.

Then, let's find its generating function, it will be $\frac{P(x)}{Q(x)} = \sum x^n \cdot p_n$. And we need to find $= \sum n \cdot p_n$. It is the value of $(\frac{P(x)}{Q(x)})'$ in the $x = 1$. You can find it with some basic knowing about differentials

Problem Tutorial: "Free Edges"

Let's take all edges, without some spanning forest. Obviously each time you can comer some leaf in black. And if you will have some cycle on the not chosen edges, all these vertices on cycle always will have degree at least two. So, the answer is the $m - n +$ number of connected components.

Problem Tutorial: "Graph Counting"

Lemma: in good graph, if you will delete all vertices, connected with each other vertex, all connected components will be cliques.

You can prove it with some facts about perfect matching, for example, with Tutte-Berge formula.

Then, let's assume that you will have x vertices connected to each other, you need to partite $2n - x$ into $x + 2$ odd parts, it is equal to the number of partitions of $n - x - 1$ in to $x + 2$ non-negative parts, it is equal to partitions of $n + 1$ into $x + 2$ parts, and sum of it for each x will be equal to the number of patitions of $n + 1$ into at least two parts, and it is equal to $P(n + 1) - 1$.

You can find number of partitions with Euler Pentagonal theorem, you need to take an inversions of some power series, you can do it with FFT.

Problem Tutorial: "Hall's Theorem"

Lemma: young diagrams are covering all k's.

Proof: write and check :)

To find an answer for fixed k, you can make a greedy like in C (it was unexpected for me), or dp with bitsets in $O(\frac{n^3 \cdot 2^n}{64})$.

Problem Tutorial: "Interesting Graph"

This property is equivalent to constraint that in each connected components at most 6 vertices. You can find chromatic polynomial in each component in $O(k \cdot 3^k + k^2)$ with dp and interpolation.

Now, you have two ways...

1) lol?.

Let's multiply all these polynomials and find the answer with multipoint evaluation.

2) ok.

Let's group all equal polynomials, and because of $P^k(x) = (P(x))^k$ you can find the answer in number of polynomials by log.

Problem Tutorial: "Jealous Split"

Let's take a partition with minimal sum of squares of sums. You can find it with Aliens trick and CHT.

Lemma: this splitting is correct.

Why? Because if you have two segments $a > b$ and $a - b > x$, you can convert them into $a - x, b + x$ and sum of squares will decrease! So, we have a contradiction! :)

Problem Tutorial: "Knowledge"

The given operations is... The presentation of group of Tetrahedral symmetry.

So, you have two generators, size 4 permutations, and you can find the answer with matrix multiplication on all permutations.