Day 1 Contest Analysis, Division AB

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Artem Vasilev, Ivan Smirnov, Filipp Rukhovich

Discover Vladivostok 2019

A. Engine called Thomas

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The graph is a *cactus*: every vertex is located on at most one simple cycle.

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Then, run any graph search algorithm (BFS or DFS) from the locomotive vertex (mark all vertices of the train as used, so you don't visit them during BFS).

If any cycle of length $\geqslant k$ is reachable, then every vertex of the graph is reachable.

Otherwise, output all vertices visited by DFS/BFS. Complexity of this solution is $O(n + m)$.

B. Broken line

Given N points on the perimeter of a convex polygon. Find a shortest polygonal chain through all these vertices.

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How to calculate $f_{i,j}$: either make a segment from point i to point $i + 1$, then take $f_{i+1,j}$, or make a segment from i to j, then take $f_{j,i+1}$.

All these calculations take $O(n^2)$ time.

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Fix any point (say, the n-th one). It can be the end vertex of a chain, or an interior one.

If it's an endvertex, check two neighbours. If it is an interior vertex, check all possible other points to be connected to the n-th one, and take the minimum.

C. Columns

Given an $n \times m$ grid, find the number of divisions into *oriented columns*.

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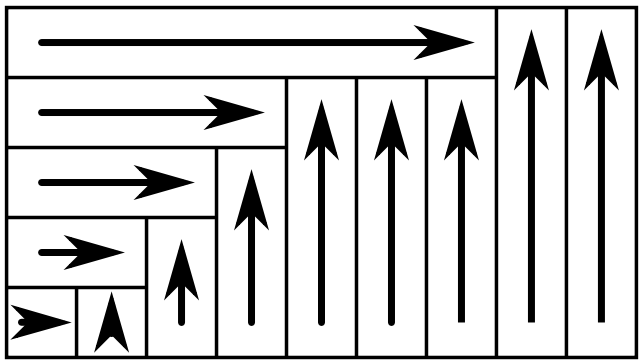
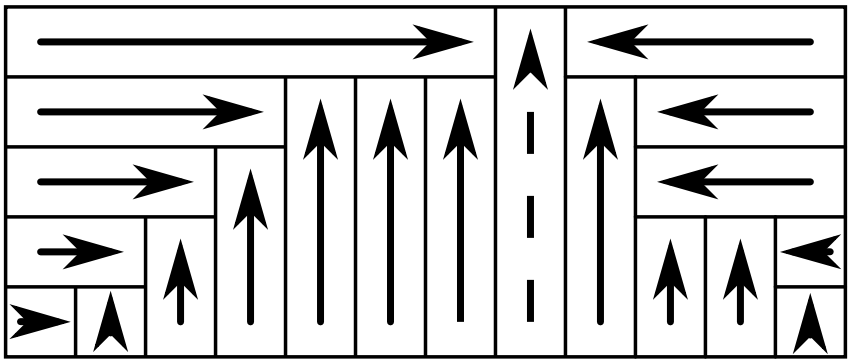
First auxiliary pattern ("2-diff"): only two different directions are allowed (e.g. "right" and "up").



The number of such patterns is C_{w+h}^{h}.

C. Columns

Second auxiliary pattern ("3-diff"): three different directions allowed (e.g. "right", "left" and "up"). At least one "up" tile must touch the top of the rectangle (dashed in the picture).

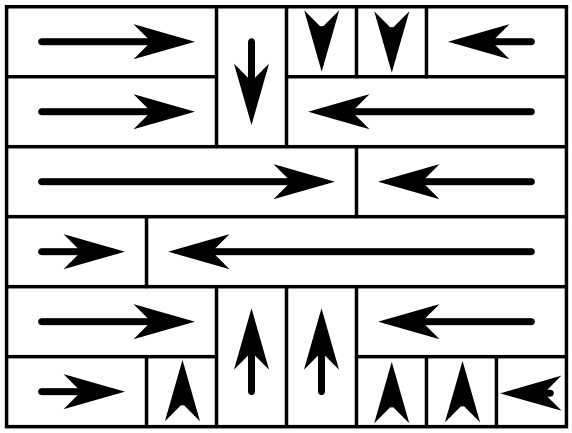


To calculate the number of such patterns, fix the position of the leftmost "up" column touching the top. Now the rectangles to the right and to the left contain exactly "2-diff" patterns.

Computation is done in $O(w)$.

C. Columns

Patterns with horizontal separator: one or several lines containing only horizontal columns.

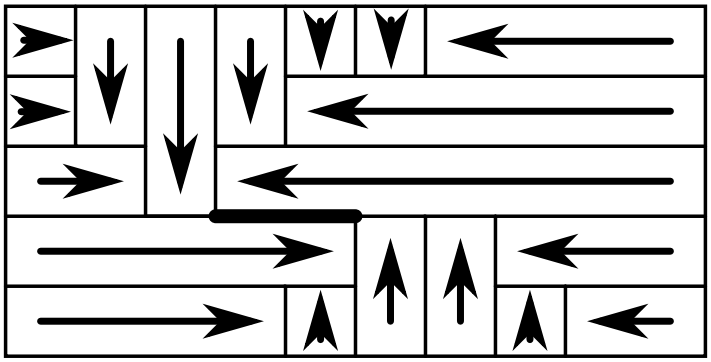


To calculate the number of ways, fix the first and the last separator line. Let there be s of them. Patterns to the top and to the bottom are "3-diff", and the number of ways to fill separator lines is $(w+1)^s$.

Computation is done in $O(h^2)$

C. Columns

General case: neither vertical nor horizontal separators exist.



C. Columns

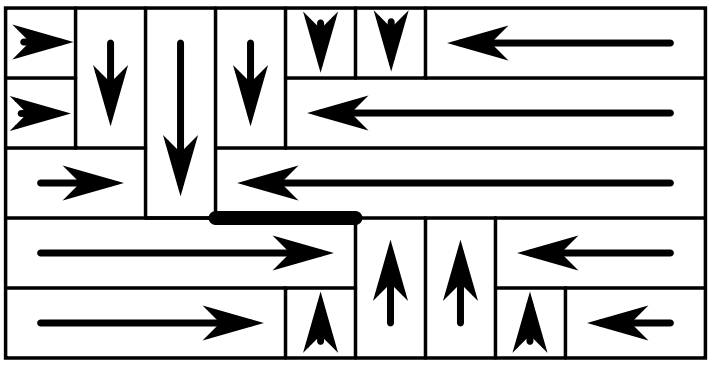
General case: neither vertical nor horizontal separators exist.

Three cases are possible.

- Two horizontal opposite-facing columns touch (depicted in the previous slide)
- Two vertical opposite-facing columns touch
- None of the above happens

We calculate the first two cases allowing the touch length to be zero and then subtract the result for the third case because we double-counted it.

C. Columns



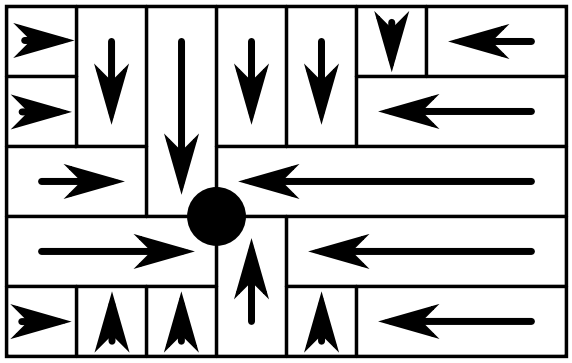
We always consider the pattern look like in the picture: two horizontal columns touch and the one facing left is at the top.

We fix the height and iterate through the position of the first "full" vertical column in the bottom part. Now the left and the right parts of the bottom parts resemble "2-diff" pattern with fixed dimensions. We use prefix sums to count all possibilities for the top part, which are also look as combinations of two "2-diff"-s.

Complexity: $O(wh)$.

C. Columns

General case where the length of the touch is zero.



We iterate through the position of the center and use "2-diff" pattern to calculate the number of ways in each of the four corners.

Complexity: $O(wh)$.

C. Columns

Outline:

- Add cases where either horizontal or vertical separator line(s) exist
- Add cases where no separator exist and two opposite-facing lines touch by a (possibly empty) segment
- Subtract cases where lines of four different directions meet at the same point

D. Zebra

Given a polygon, and a zebra-like coloring of a plane, find the white area inside the polygon.

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We will select trapezoids that have bases parallel to zebra lines. Areas of intersections of whites line with this trapezoid form an arithmetic progression, sum of which can be easily found.

We consider n trapezoids, each in $O(1)$ time.

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Fact 1: after 2^k-th stage, the zeroth element is equal to XOR of elements with numbers 0 and 2^k (can be proved by induction).

Fact 2: after the i-th stage the first element is equal to XOR of all elements j where j is a submask of i:

$result_i = \bigoplus_{j \subset i} input_j$ (consequence of Fact 1).

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It is possible to calculate this array in $O(n \log n)$ time using an algorithm from Fast Subset Convolution.

Might require some optimizations, since time limit is really tight.

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But how to solve it if you don't know anything about Fast Subset Convolution?

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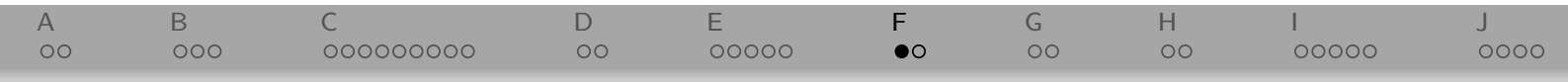
Then, suppose that $n = 2^l$ for some integer l (if not, add some number of zeroes to the end of 0th row). Then, use divide-and-conquer technology:

- calculate first, "upper", half of 0th column and 2^{l-1}-th column recursively;
- calculate second half of 0th column as bitwise XOR of first half of 0th column and 2^{l-1}-th column (consequence of Fact 1).

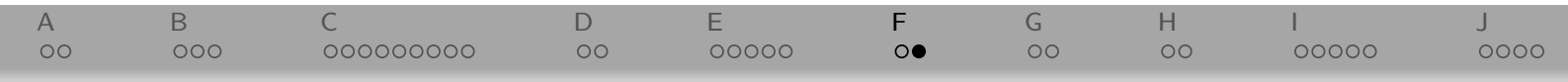
G. Pluses and minuses

The complexity of such a method is $O(n \ log \ n)$; but if you store columns in std::bitset (or your custom version) then it will work in $O(n \ log \ n/32)$ which should be fast enough even for $n = 5 * 10^6$)))



F.Towers of Hanoi

Find the minimum number of steps to solve the Hanoi Towers puzzle from the given starting configuration.



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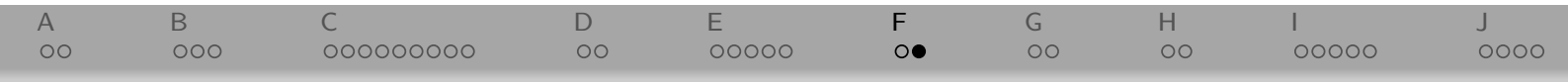
Iterate from the largest disks to smallest ones. Keep track of the current target rod.



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Iterate from the largest disks to smallest ones. Keep track of the current target rod.

If the current disk is on the target rod, skip it. Otherwise, you have to move everything to the third rod, then move the current disk to target rod, then move all others into the target rod.

This takes $2^x + y$ operations, where x is the number of current disk, and y is the cost of moving all disks to a new target rod.

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In other words, just add 2^x each time the current disk is not on target rod, then switch targets. $O(n)$ time.

G. Interesting sequence

Find p-th occurrence of n in the given sequence (a_i).

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First of all, notice that a_i is a number of maximal by inclusion series of equal consecutive bits in binary representation. For example, $a_{13} = a_{1101_2} = 3$, $a_8 = a_{1000_2} = 2$, $a_{15} = a_{1111_2} = 1$.

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Then, let $dp[l][k]$, $1 \leqslant k \leqslant l \leqslant 63$, is a number of 0,1-sequences of length l with exactly k series of equal consecutive bits (Why 63? Because the bit length of answer is not more than 63).

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Obviously, $dp[l+1][k] = dp[l][k] + dp[l][k+1]$ (first bit can be equal to second one or not). Using this dynamics, one can find p-th 0,1-sequence with n series starting from one by the same way as p-th combination of k elements of set $\{1, 2, 3, 4, \ldots, n\}$ can be found.

H. Transformation on numbers

Find the number of operations of adding 10^y and changing a digit to transform one given integer into another.

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When you process all digits, the answer for a state with no carry is the final result.

I. Kingdom division

Split a tree into three connected components A, B and C such that $|A| = |B|$. Maximize $|A|$.

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If u is not a root, then everything above it is either A, or $A \cup B$. In the first case you have to check if u has a subtree of size $n - size_u$, in the second case you have to check whether the $A \cup B$ part can be split into two equal subtrees.

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If u is a root, you need to find two subtrees of equal sizes. This all can be done, if you store a multiset of all subtree sizes for every vertex and merge them small-to-large.

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However, there is a way to solve it in $O(n \log n)$ without mergeable multisets! All you need is to calculate $size_u$ and binary lifts for each vertex u. Also let $cnt[k]$, $k = 1, \ldots, n$, be a number of such a vertices u that $size_u = k$.

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Then, to get three components, we should erase two edges. They may be ancestor and descendant - or not. Consider these two cases.

Suppose that we want to erase an edge from known vertex u to its parent, and also we want to erase and edge from some unknown vertex v to its parent in such a way that v should be ancestor of v.

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Obviously, $size_v$ should be equal to $2 * size_u$, $n - size_u$ or $n - (n - size_u)/2$; existing of such a v on the path from u to root can be established in $O(log\ n)$ for each u using binary search on binary lifts.

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To do that, $size_v$ should be equal to $size_u$, $n - 2 * size_u$ or $(n - (n - size_u))/2$. Suppose that we are checking some value $S \geqslant size_u$ to be $size_v$.

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If $size_u = S$ then we should just check whether $cnt[size_u] \geqslant 2$ or not.

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Otherwise, we should see on $cnt[S]$ too. If $cnt[S] = 0$ then v does not exist; if $cnt[S] \geqslant 2$ then good v definitely exists; if $cnt[S] = 1$ then we should check whether the corresponding v is an ancestor of u or not; it can be done using the same binary search (or arrays *tin* and *tout*)))).

J. Domino sets

Given a set of m-tiles of dominoes; each one is of one of v types. We can drill new holes in any tile and are to calculate maximum number of full m-sets we can make.

We will solve the problem using binary search by answer. So, given positive integer k, is it possible to make at least k m-piles?

J. Domino sets

Suppose for each possible type (i, j) of tile that $i \leqslant j$. Before binary search, sort all given types by i, and in case of equal i-s - by j.

During binary search, we should transform tiles into at least k $(0, 0)$-tiles, k $(0, 1)$-tiles, ..., (m, m)-tiles.

Suppose for beginning that we should build only tiles with $i = 0$. Then, create a stack ST of pairs (type of tile, number of such tiles). After that, iterate over all j from 0 to m and try to make k $(0, j)$-tiles by the following way:

J. Domino sets

- if there are z $(0, j)$-tiles in the initial tile, add pair $((0, j), z)$ into ST;
- after that, ST contains all tiles which can be transformed into $(0, j)$-tiles. If ST contains less than k elements then we cannot form k m-tiles; otherwise, it's optimal to do it from top k elements of ST, and we should remove them from ST.

It works in $O(m + v)$ time; but not that if input does not contains, for example, tiles $(0, l), (0, l + 1), \ldots, (0, r)$ for some segment $[l, r]$ then iterations $l, l + 1, \ldots, r$, can be performed as one iteration of removing $k * (r - l + 1)$ tiles from ST. This idea reduces the complexity to $O(v)$.

J. Domino sets

Then, learn how to solve a problem without assumption about $i = 0$.

First of all, tile with $i = 0$ can be made only from tiles with $i = 0$; try to build it by the way described above.

After that, ST may contain some tiles we can use later; for each such tile, increase i by one (and j if it is needed) and return it back to the initial set. Then repeat the algorithm for $i = 1, 2, \ldots, m$. It gives us a full check in $O(vm)$ time.

It is possible to reduce the time to $O(v^2)$ by the way similar to optimization of $[l, r]$. So, the total complexity of the solution is $O(v^2 * log\ MAX\_POSSIBLE\_ANSWER)$.