Long Contest Editorial

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A. As Easy As Possible

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It is easy to answer a single query in $O(n)$ time.
In fact, we will find the longest prefix of the infinite string `(easy)*` (infinite repeats of `easy`) which is a subsequence of the substring t: start with the empty prefix, iterate over characters of t and increase the length of the prefix whenever the current character of t matches the next character of `(easy)*`.

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Since `(easy)*` is periodical, it suffices to know the next character of `easy` to match with. Thus, we precompute $a_{k,i,j}$ — how much the prefix can be extended inside the substring $s[i \ldots i + 2^k)$ if the current character to match is `easy`[j].

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Time and memory needed to precompute $O(nm \log n)$, where $n = |s|$, $m = |\texttt{easy}|$. It is then possible to answer a query in $O(\log n)$ time, for the total $O((q + nm) \log n)$ time complexity.

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This hints at the recursive approach: divide all numbers into two groups according to their k'th digit, solve recursively for these groups, then add minimal possible edge between the groups.

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When we descend into a child of the first vertex, choose a child of the second vertex that minimizes XOR in the current digit (if possible).

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We will use a greedy approach. Do parallel DFS of the subtrees. When we descend into a child of the first vertex, choose a child of the second vertex that minimizes XOR in the current digit (if possible). For each pair of leaves reached, try to improve the answer with their XOR. Clearly, an optimal answer will be found at some leaves pair.

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Each vertex of the trie is present in at most $k + 1$ subtrees, so total complexity is $O(k^2 n)$ since there are $O(kn)$ vertices in the trie.

C. Coprime Heaven

We are given $k \leqslant 4$ numbers l_i. Distribute numbers from 1 to $n = \sum l_i$ into circles of lengths $l_1, \ldots, l_k$ such that each pair of adjacent numbers is coprime.

C. Coprime Heaven

Clearly, even numbers cannot be adjacent. A circle of length $l > 1$ can include at most $\lfloor l/2 \rfloor$ even numbers (or at most 1 if $l = 1$). If there are more even numbers than we can include in all the circles, then no answer exists.

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Can be done by a long and tedious case analysis and constructions. Key insight: it is convenient to include runs of adjacent numbers into circles, then only ends of the runs should be checked for coprimeness.

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Always finds a solution (at least, on the jury test cases), and works very fast too (which hints that there are many solutions of similar form).
An unchecked conjecture: starting from trivial configuration (put smallest numbers into first circle, the next into the second circle, and so on) and performing local optimization/simulated annealing should work too.

D. Drawing Hell

A set of n points is given in the plane. Two players play a game, a move is to connect two points with a segment if the segment does not contain other points and does not intersect previously drawn segments. Determine the winner of the game if the player who is unable to make a move loses and both players act optimally.

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Proof

The first case is obvious. For the second case, we notice that any final configuration is a triangulation of the initial set (that is, every face of the resulting planar graph is a triangle). Euler's formula $V-E+F=2$ implies that any triangulation of a non-collinear set has $3n-3-c$ edges.

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The number of test cases is large, thus one should use an $O(n \log n)$-time algorithm for convex hull.

E. Easiest game

Process several queries: for an $m \times n$ board count number of pairs (s, t) with $1 \leqslant s \leqslant t \leqslant \max(m, n)$ such that an (s, t)-knight can visit all board cells.

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Both are hard.

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Necessity: if $g = GCD(t+s, t-s) > 1$, then $x+y$ and $x-y$ are always the same modulo g, thus not all cells are reachable from each other.

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Sufficiency: ???

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Also, denote $f'(n, m)$ the number of pairs with same restrictions except for the last one (same parity).

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The difference is, if k is even then we shouldn't care about same parity of a and b inside $f(\lfloor n/k \rfloor, \lfloor m/k \rfloor)$, so in this case we replace it with $f'(\lfloor n/k \rfloor, \lfloor m/k \rfloor)$.

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To optimize further, note that there are $O(\sqrt{n} + \sqrt{m})$ values of k such that $\lfloor n/k \rfloor$ or $\lfloor m/k \rfloor$ change.
Thus, the solution can be optimized to $O(\sqrt{n} + \sqrt{m})$ per test by processing segments of k where $\lfloor n/k \rfloor$ and $\lfloor m/k \rfloor$ are fixed.

F. Fibonacci of Fibonacci

Find $F_{F_n} \bmod 20160519$.

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For each integer m Fibonacci numbers modulo m eventually loop, that is, there exists $p > 0$ such that $F_n \equiv F_{n+p} \pmod{m}$ for each n.

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Since there are many queries to answer, one should use matrix exponentiation to find the answer in $O(\log n)$ per query:

Proposition

$$\begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} F_1 \\ F_0 \end{pmatrix}$$

G. Global Warming

Given two convex polygons — the planet and the moon, and the radiation direction from the sun, find the total heat from the sun assuming that the light can reflect from the moon.

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The model of effective heat absorption in the statement is somewhat obscure. However, it helps to think of it this way: assume that the planet is illuminated by a single strip of light. Then the resulting amount of heat from this strip of light is the *width* of the strip part that falls on the surface of the planet.

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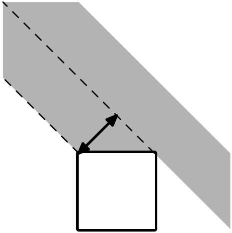

For example, on this picture the equivalent length is equal to half the square diagonal length.

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For the polygon we can similarly find border rays which fall on its surface. It is evident that their c's (denote them $\underline{c'}$ and $\overline{c'}$) are simply the extremal values of $ax + by$ over all vertices of polygon.
The equivalent length of the illuminated part is then the length of the $[\underline{c}; \overline{c}] \cap [\underline{c'}; \overline{c'}]$ range.

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One possible approach is based on the fact that $ax + by$ function is unimodal (with a unique local extremum) on a convex curve such that its, say, x coordinate does not decrease. Hence, the $ax + by$ function is unimodal on lower and upper halves of the polygon, so the ternary search can be applied to each of them.

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This reasoning yields a way to find extremums of $ax + by$ over convex polygon in $O(\log n)$ time.

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G. Global Warming

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The total complexity is $O(n + m \log n)$.

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where p is the *base*, and m is the *modulo*.

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Applying this reccurence with $n_1 = 1$, one can obtain a $O(nm\alpha)$ solution (since $d_{1,h} = 1$ iff h is a hash of a single letter).

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To speed this up, note that the described recurrence can be computed as coefficients of the polynomial product $P(x) \times Q(x)$, where
$P(x) = \sum_{i=0}^{m-1} d_{n_1,i} x^i$, and $Q(x) = \sum_{i=0}^{m-1} d_{n_2,i} x^{(p^{n_1} i) \bmod m}$.

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Now, suppose that we know the numbers $d_{n,h}$. Let
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- and for an even n find the answers for $n/2$ and use the method above.

We obtain a solution that does $O(\log n)$ m-degree polynomial multiplications.

H. Hash Collision

Of course, to speed up we should use FFT to multiply polynomials. Since the answer should be found modulo $10^6 + 3$, the `double` precision should be enough. The resulting complexity is $O(m \log m \log n)$.

I. Increasing or Decreasing

Process many queries of the form: count the numbers inside the range $[L; R]$ which decimal representations are monotonous (that is, teh digits are either increasing or decreasing).

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First approach: compute DP of sort "how many non-increasing/non-decreasing l-digit prefixes of n-digit numbers exist such that they are already less than/still equal to corresponding part of R/L" (this is messy to code and will probably need some optimization).

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Second approach: generate all suitable numbers and note that there are less than 20 million of them. Hence, each query is simply several binary searches in precomputed lists.

J. Just Convolution

Given $a_0, \ldots, a_{n-1}$ and $b_0, \ldots, b_{n-1}$ — random permutations of $\{0, \ldots, n-1\}$, find $c_0, \ldots, c_{n-1}$, where

$$c_k = \max_{i=0}^{n-1}(a_i + b_{(k-i) \bmod n})$$

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For convenience we will replace max *with* min *in the above formula, which does not change the problem much.*

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Depending on K, how many elements on average will require a brute-force?

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Proposition

Suppose $K > \sqrt{n}$. For a given index k, the probability that c_k will require a brute-force is at most $e^{-K^2/2n+O(K^3/n^2)}$.

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Proceeding this way, we conclude that the resulting probability is

$$\frac{(n-K)^K}{n!/(n-K)!} = \frac{(1-K/n)^K}{(1-1/n)\dots(1-(K-1)/n)} = \dots$$

J. Just Convolution

Proof (continued)

$$\ldots = \exp(K \ln(1 - K/n) - \ln(1 - 1/n) - \ldots - \ln(1 - (K-1)/n))$$

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For the worst case $n = 2 \cdot 10^5$, choosing $K = 2000$ will produce hardly a hundred of unprocessed elements.